Introduction to
Coherent Synchrotron Radiation

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COHERENT SYNCHROTRON RADIATION*

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ABSTRACT

A simple model consisting of a distribution of charges constrained to move on a ring is the basis of an investigation of coherent synchrotron radiation. The radiation produced as a result of a nonrandom particle distribution on the ring is examined from the viewpoint of the interaction of individual particles with the total electric field of the system. A linear stability analysis shows that, under reasonable conditions, a uniform distribution of particles is unstable to clumping. The model is applied to pulsars, in which the high brightness temperatures suggest that a cooperative emission mechanism is responsible for the radiofrequency radiation. The application to circular accelerators and storage rings is discussed briefly.

c) Circular Accelerators and Storage Rings

A one-dimensional continuity equation has been used in the preceding analysis. This will not generally be a good approximation for the motions of particles in accelerators or storage rings. Thus the analysis of the instability is not applicable in general. Even for cases where the approximation is not too bad, the conditions for instability, and for negligible interference caused by the energy spread in the beam, are not easily satisfied. Under most conditions the growth rate $s/\omega_0 \ll 1$. In addition, the presence of metal surfaces near the beam would probably tend to damp the instability. Thus it seems unlikely that the instability will be important for high-energy particle machines.

ORIGIN OF COHERENCE in an ELECTRON BUNCH

- Define $I_e(\omega) \equiv \left|E_e(\omega)\right|^2$ = radiation intensity from 1 electron.
- Assume all electrons are accelerated the same way.

$$\boldsymbol{E}_{tot}(\omega) = \sum_{j=1}^{N} E_j(\omega) e^{i(k\boldsymbol{n}_j \bullet \boldsymbol{r}_j - \omega t)} \;;$$

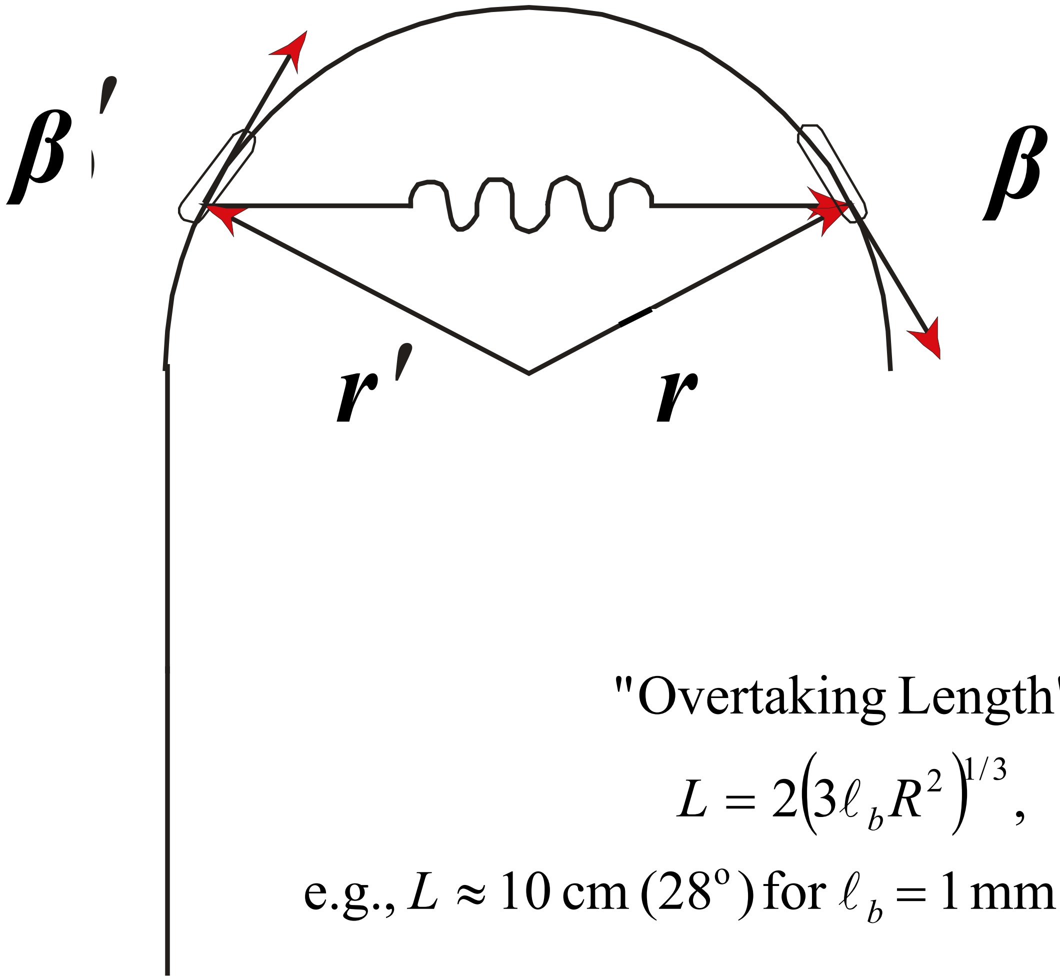
$$I(\omega) = \left|\boldsymbol{E}_{tot}(\omega)\right|^2 = \left|\sum_{j=1}^{N} E_j(\omega) e^{ik\boldsymbol{n}\bullet\boldsymbol{r}}\right|^2 = \sum_{j=1}^{N}\left|E_j(\omega)\right|^2 + \sum_{\substack{j=1\\ j\neq m}}^{N}\sum_{m=1}^{N} E_j(\omega)E_m^*(\omega) e^{ik\boldsymbol{n}\bullet(\boldsymbol{r}_j - \boldsymbol{r}_m)}$$

$$= NI_e(\omega) + I_e(\omega)\sum_{\substack{j=1\\ j\neq m}}^{N}\sum_{m=1}^{N} e^{ik\boldsymbol{n}\bullet(\boldsymbol{r}_j - \boldsymbol{r}_m)} \;.$$

There is little phase difference in emitted fields for wavelengths λ large compared to the bunch size:

$$\Rightarrow I(\omega) \approx NI_e(\omega) + N(N-1)I_e(\omega) \Rightarrow I(\omega) \approx N^2 I_e(\omega).$$

BUNCH SELF-INTERACTION VIA CSR



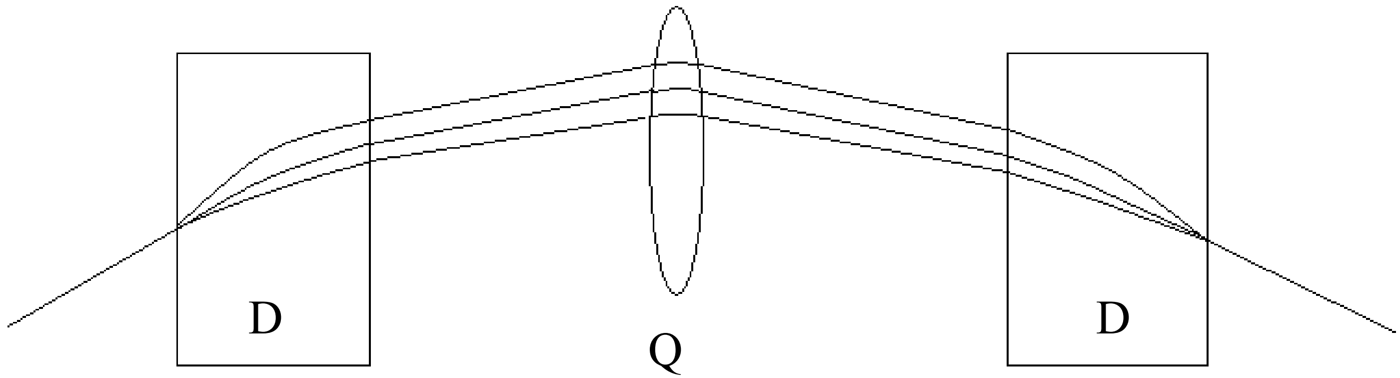
"Overtaking Length":

$$L = 2\left(3\ell_b R^2\right)^{1/3},$$

e.g., $L \approx 10$ cm (28°) for $\ell_b = 1$ mm and $R = 20$ cm.

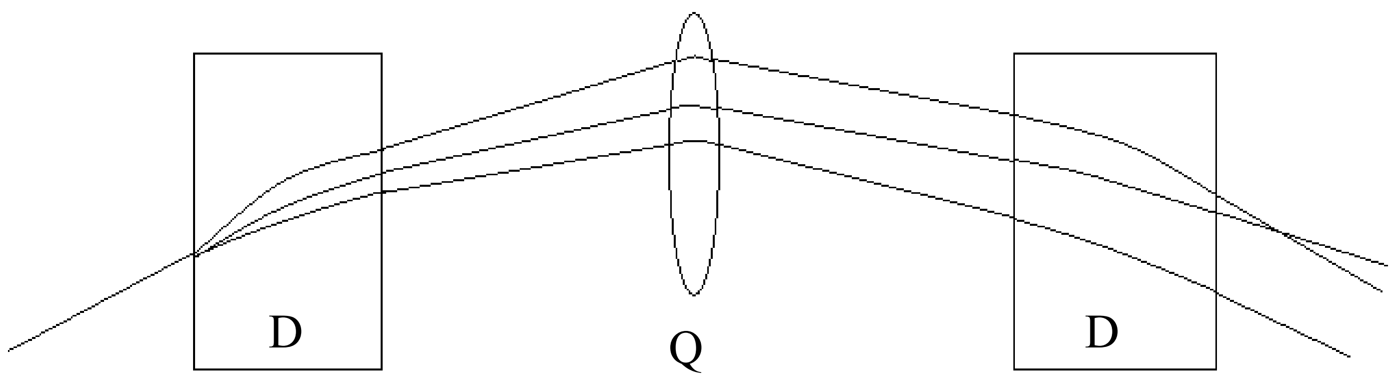
INDUCED ENERGY SPREAD AND EMITTANCE GROWTH

Example: Achromatic bend through angle θ; ΔE = energy spread induced ***in the bend***.



$\Delta E = 0$ (ideal case)

$\Delta\theta = 0$



$\Delta E \neq 0$

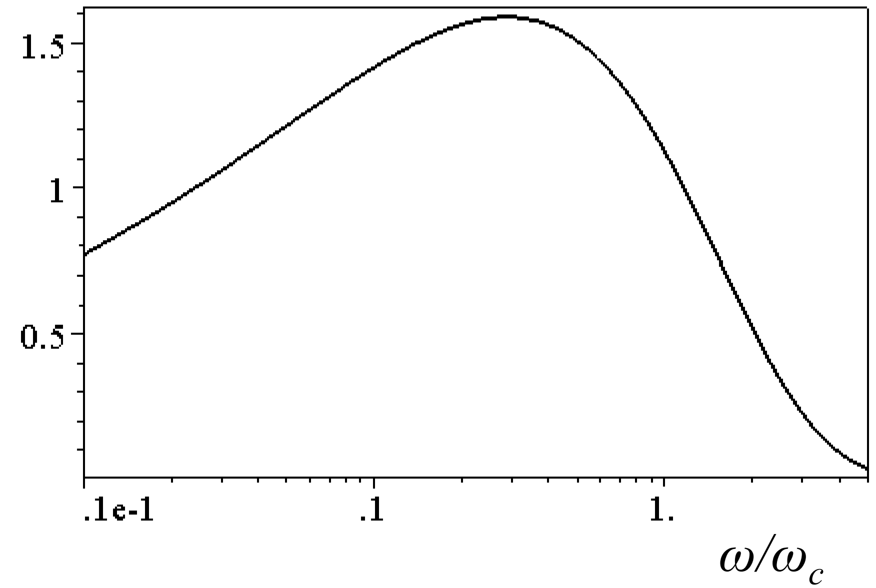
$\Delta\theta \neq 0$

SYNCHROTRON RADIATION SPECTRUM FOR CHARGE IN CIRCULAR MOTION

For large γ:

$$\frac{c}{e^2\gamma}\frac{dI}{d\omega} = \sqrt{3}\,\frac{\omega}{\omega_c}\int_{\frac{\omega}{\omega_c}}^{\infty} dx\, K_{5/3}(x)$$

$$\frac{c}{e^2\gamma}\frac{dI}{d\omega}$$



$$\omega_c = \frac{3}{2}\gamma^3\frac{c}{R} \leftrightarrow \lambda_c = \frac{4\pi}{3}\frac{R}{\gamma^3};$$

For $\lambda \geq \sigma_z > \lambda_c$ (CSR regime),

$$\frac{dI}{d\omega} \approx 3.25\frac{e^2}{c}\left(\frac{\omega R}{c}\right)^{1/3} \Rightarrow \gamma - \text{independent.}$$

Spread in angle:

$$\theta_c \approx \left(\frac{3}{2\pi}\frac{\lambda}{R}\right)^{1/3}, \text{ e.g., } \approx 8^o$$

for $\lambda = 1$ mm, $R = 20$ cm.

HAMILTONIAN FORMULATION

Single-particle hamiltonian:

$$\boxed{\begin{aligned} H &= c\sqrt{\left(\boldsymbol{P}-\frac{e}{c}\boldsymbol{A}\right)^2+(mc)^2} + e\varphi, \\ \begin{bmatrix}\varphi(r,t)\\ A(\boldsymbol{r},t)\end{bmatrix} &= e\int d\boldsymbol{r}'dt' \frac{\delta\left(t-t'-\frac{1}{c}\left|\boldsymbol{r}-\boldsymbol{r}'\right|\right)}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|}\begin{bmatrix}1\\ \boldsymbol{\beta}'\end{bmatrix} n(\vec{\boldsymbol{r}}',t').\end{aligned}}$$

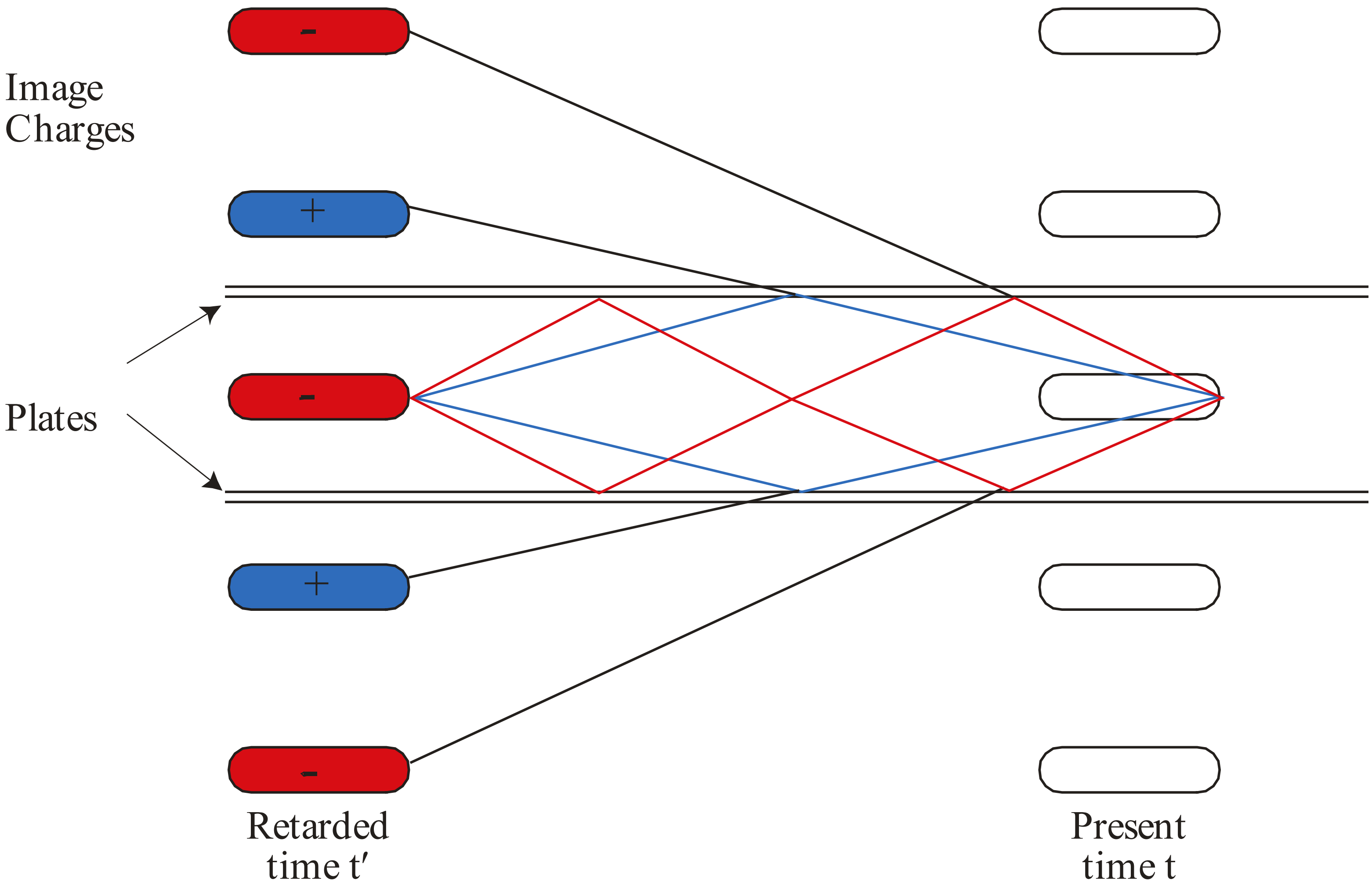
Simple, yet *excruciatingly complex*!

Change in single-particle energy: $\dfrac{dH}{dt} = e\dfrac{dV}{dt}$; $\quad V=\varphi-\boldsymbol{\beta}\cdot\boldsymbol{A}.$

Change in kinetic single-particle energy: $mc^2\dfrac{d\gamma}{dt} = -e\dfrac{d\varphi}{dt} + e\dfrac{\partial V}{\partial t}.$

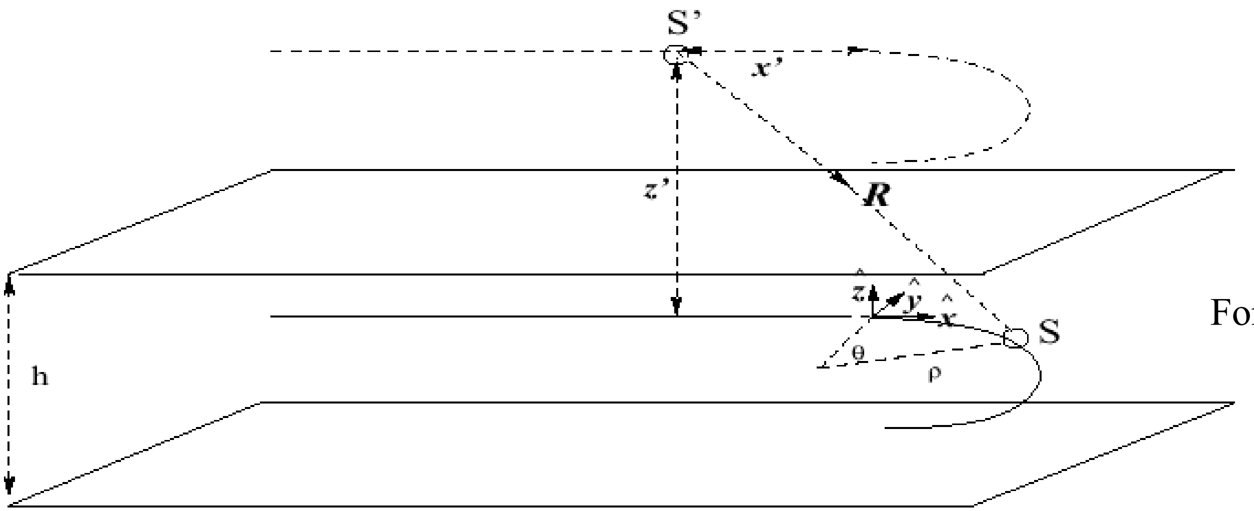
Rate of work done by bunch against itself: $P(t) = -mc^2\displaystyle\int d\boldsymbol{r}\,\frac{d\gamma(\boldsymbol{r},t)}{dt}n(\boldsymbol{r},t).$

BUNCH SELF-INTERACTION IN VACUUM CHAMBER



TRANSIENT POWER LOSS OF A BUNCH ENTERING A CIRCLE FROM A STRAIGHT PATH

[R. Li, C.L. Bohn, J.J. Bisognano, Proc. SPIE, Coherent Electron-Beam X-Ray Sources, SPIE Vol. 3154, 223 (1997).]

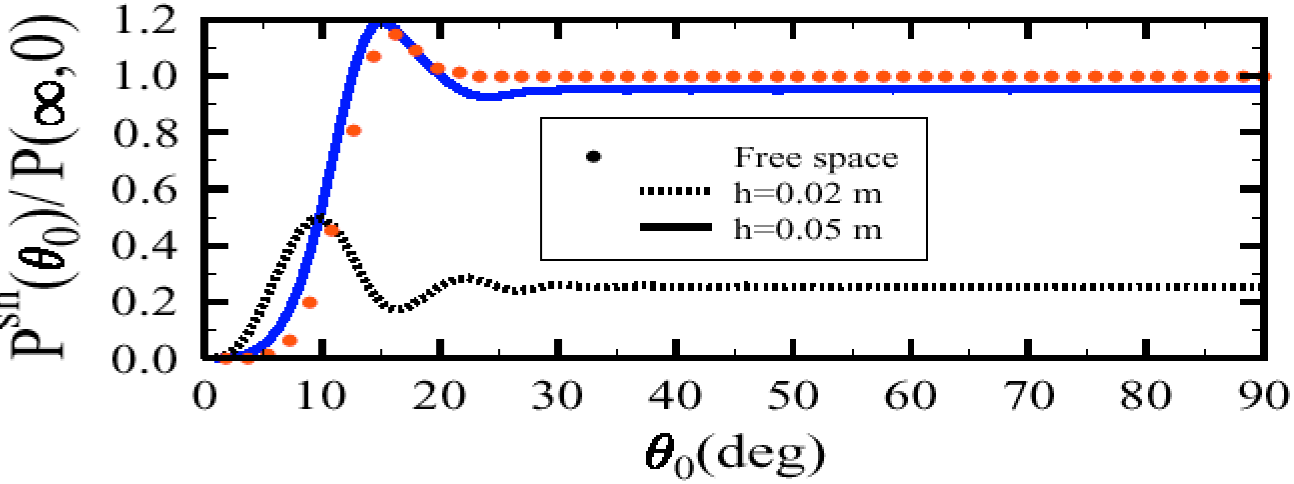


Shielding Factor:

$$\eta = \sqrt{\frac{2}{3}}\left(\frac{\pi R}{h}\right)^{3/2}\frac{\sigma_z}{R}.$$

$\eta \geq 1 \Rightarrow$ strong shielding.

For $\sigma_z = 1$ mm, $R = 1$ m, and $h = 2$ cm, $\eta = 1.6$.



$R = 1$ m,
$E = 40$ MeV,
Gaussian bunch with $\sigma_z = 1$ mm.

CANCELLATION OF LOCAL INTERACTION

[R. Li, Proc. 2002 EPAC (submitted).]

Equation of Transverse Motion:

$$\frac{d}{dt}\left(\gamma m \frac{dr}{dt}\right) - \gamma_{design} mc^2\left(\frac{\beta_\theta^2}{r} - \frac{\beta_\theta \beta_{design}}{R}\right) =$$

$$\beta_\theta^2 \frac{\left(\gamma_{initial} mc^2 + e\phi_{initial}\right) - \gamma_{design} mc^2}{r}$$

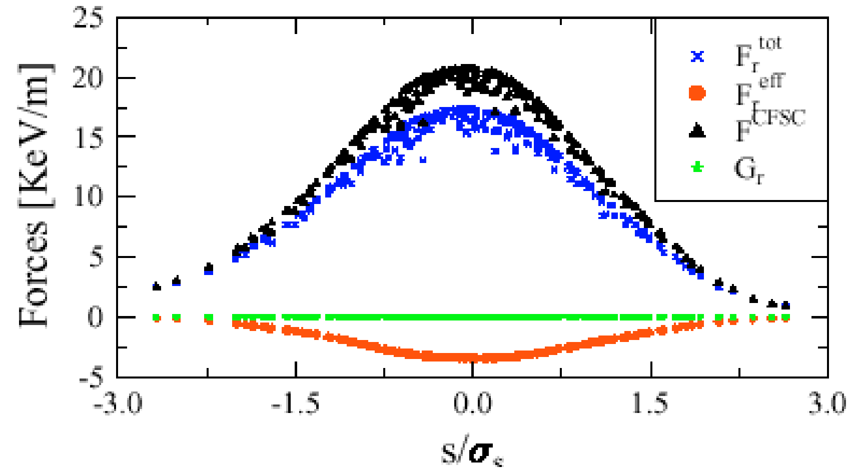
$$+ e\frac{\beta_\theta^2}{r}\int_{t_0}^{t} dt' \frac{\partial(\phi - \boldsymbol{\beta}\cdot\boldsymbol{A})}{\partial t'}$$

$$- e\frac{\partial(\phi - \boldsymbol{\beta}\cdot\boldsymbol{A})}{\partial r} - \frac{e}{c}\frac{dA_r}{dt} \qquad \equiv F_r^{eff}$$

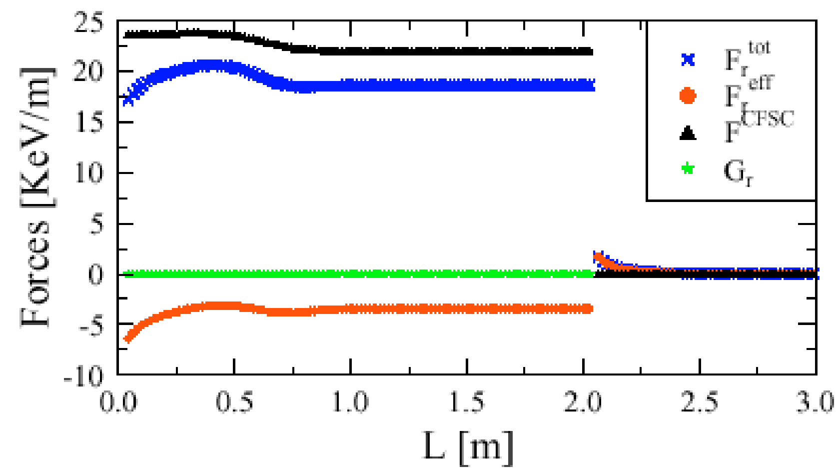
$$- e\beta_\theta \frac{\beta_\theta \phi - A_\theta}{r} \qquad \equiv G_r$$

$$= \text{“}F_{NSCF}\text{”} + F_{CSCF}$$

$$F_r^{tot} \equiv F_r^{eff} + F_{CSCF}$$

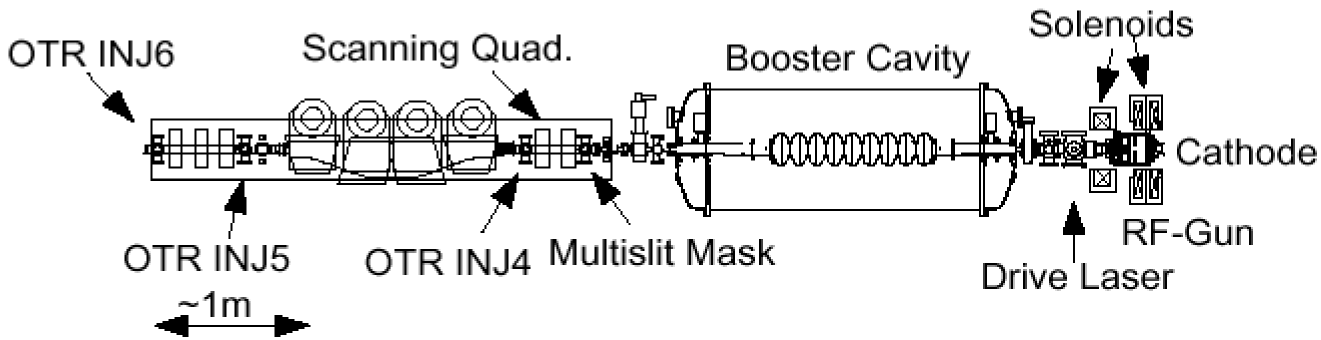


Forces across a gaussian bunch.



Forces on one particle vs. path length.

PROCEDURE FOR BUNCH COMPRESSION



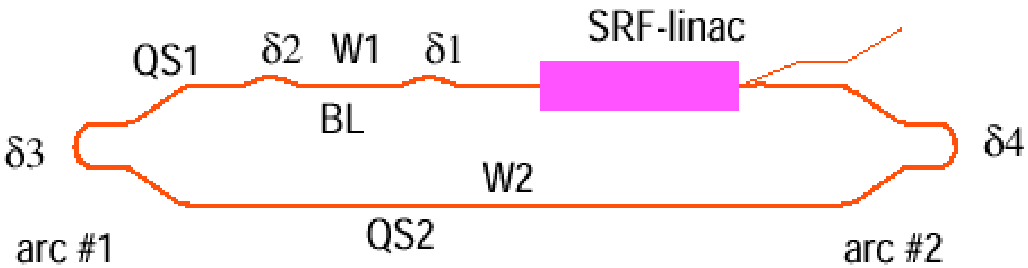
DESY TTF, Fermilab Photoinjector

- Impart correlated energy spread across the bunch
  - accelerate off of the rf crest
  - particles in tail pick up more energy than particles in head

- Impart energy-dependent path lengths
  - use dipole magnetic fields
  - high-energy tail catches up with low-energy head
  - net effect: rotation of longitudinal phase space (*or is it???*)

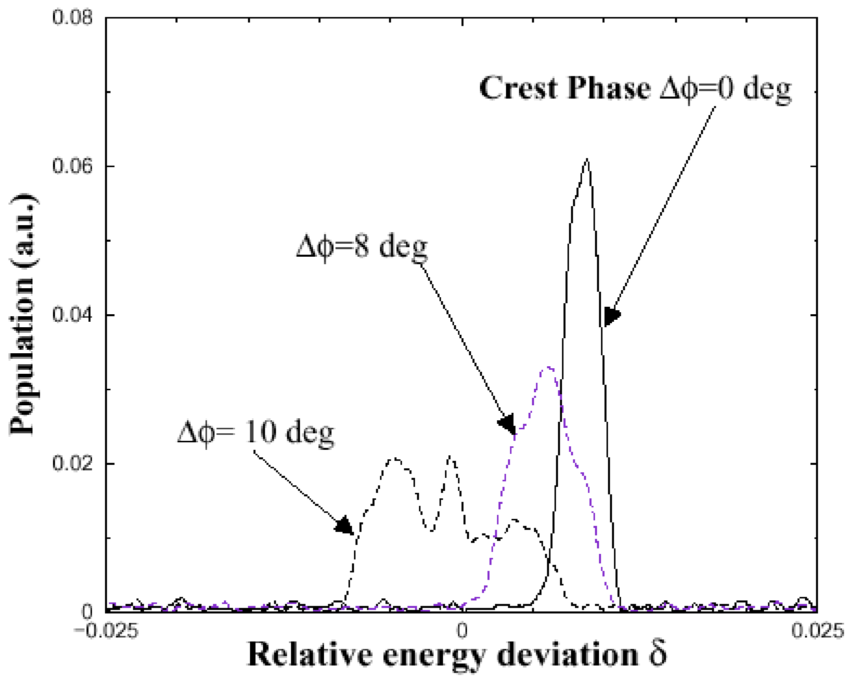
EARLY EXPERIMENTS WITH JEFFERSON LAB'S FEL

[Ph. Piot, *et al*., Proc. 2000 EPAC, 1546 (2000).]

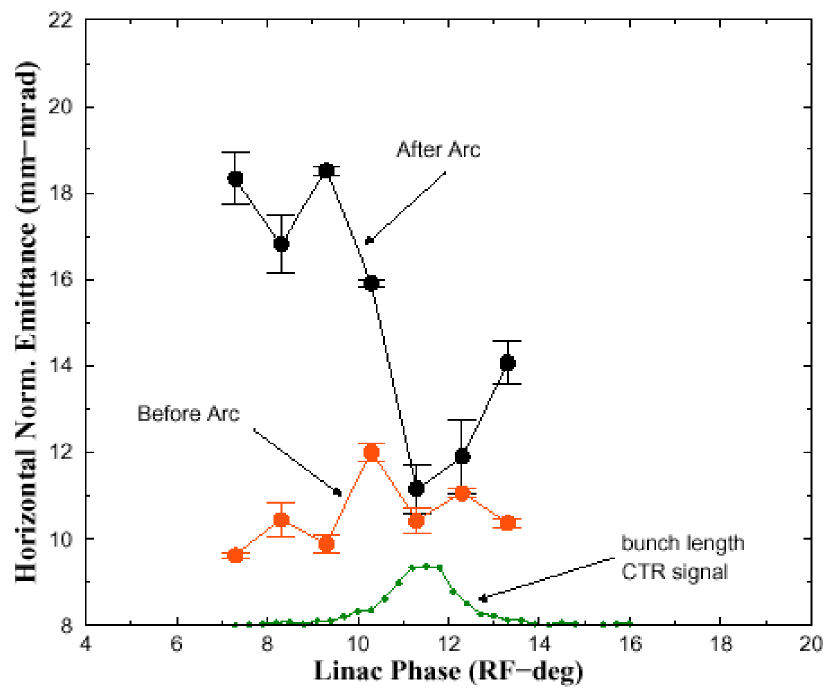


(Measurements made in March 1999.)

~0.04 nC bunches at 40 MeV



Fine structure at δ2 was enhanced at larger bunch charge.

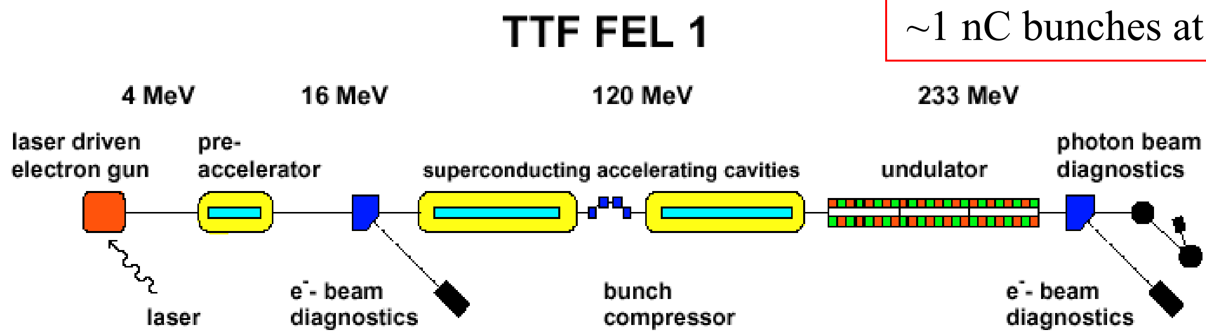


Emittance grew as longitudinal waist was moved toward the arc #1 entrance.

OBSERVATION OF PHASE-SPACE FRAGMENTATION

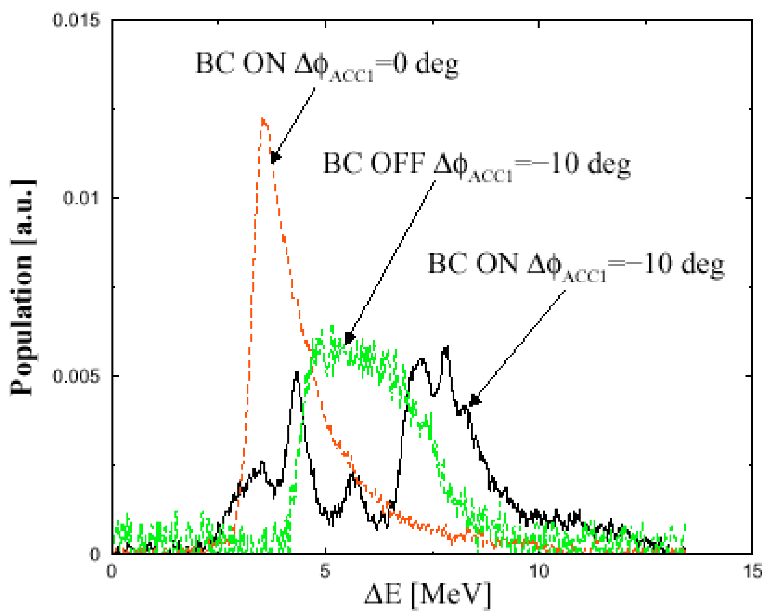
[M. Huening, *et al.*, *NIM* **A475**, 348 (2001).]

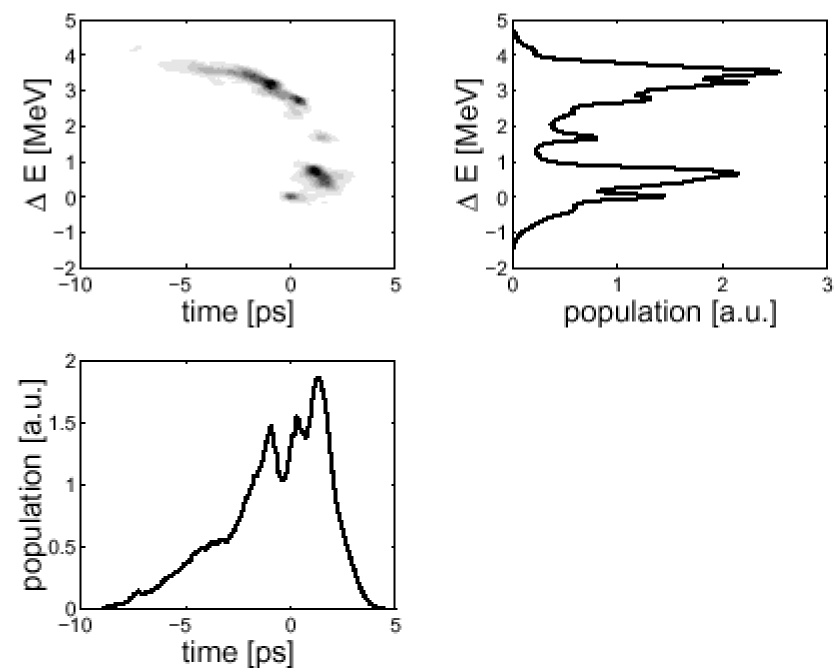
~1 nC bunches at 135 MeV



Fragmentation (only) near max. compression:

Tomographically reconstructed phase space:

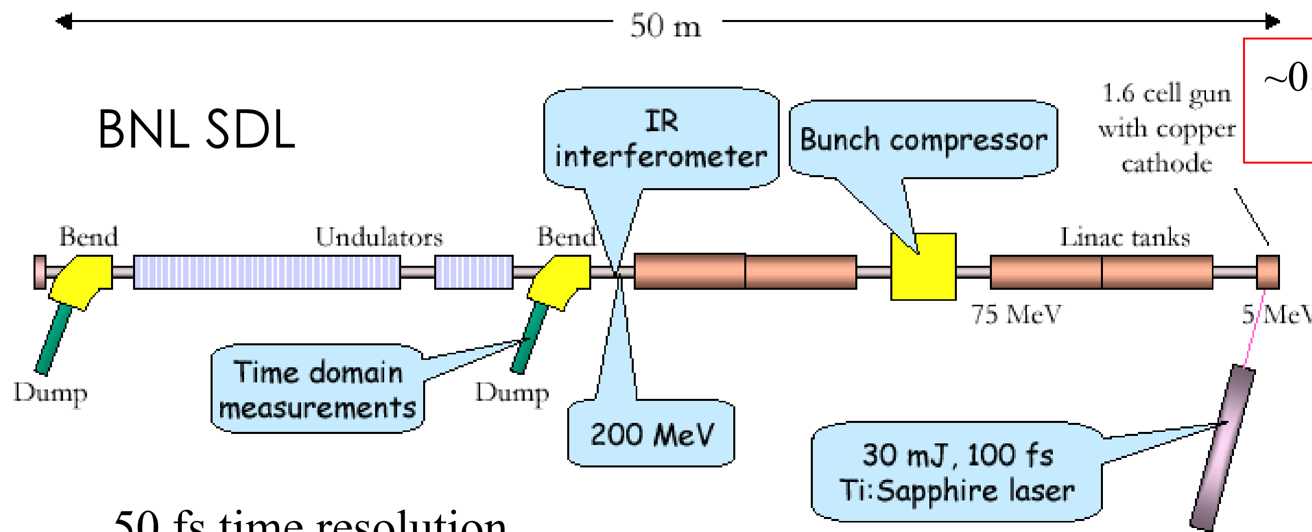




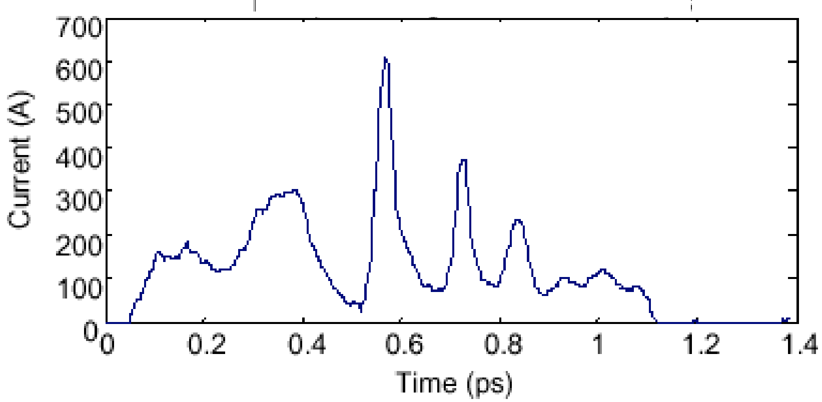
PRONOUNCED MICROBUNCHING

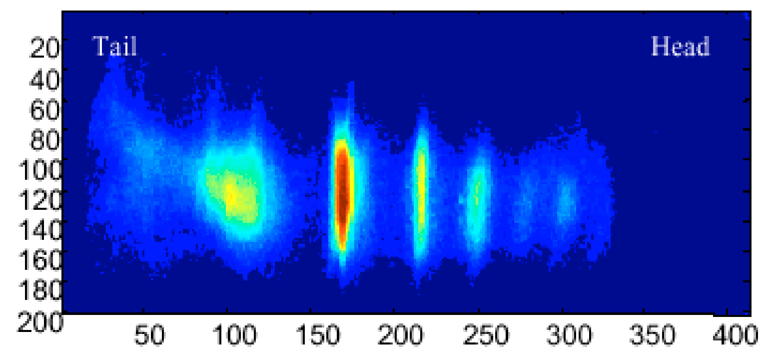
[W.S. Graves, et al., Proc. 2001 PAC, 2224 (2001)]

~0.25 nC bunches at 75 MeV



50 fs time resolution



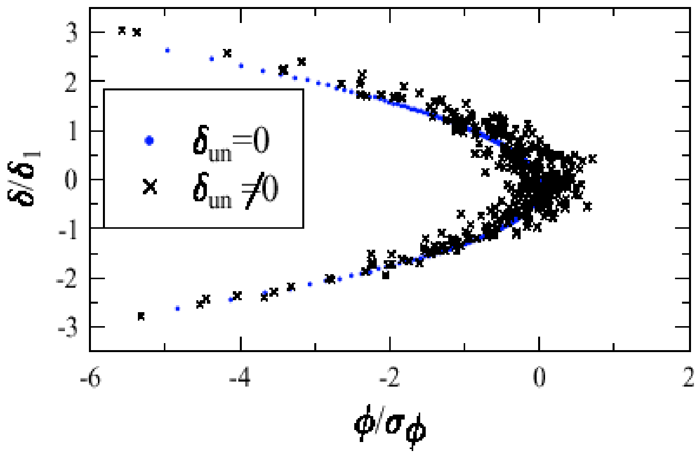


Strong compression.

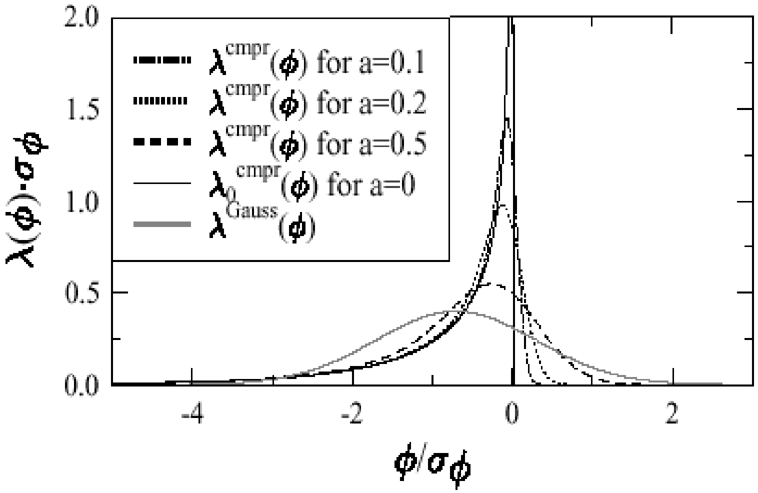
Modulations seen to be sensitive to the phase-matching angle of the laser-doubling crystals. But, there is an open question on the role, if any, of surface-roughness wakefield.

ROLE OF RF CURVATURE

[R. Li, Proc. 2000 EPAC, 1312 (2000)]

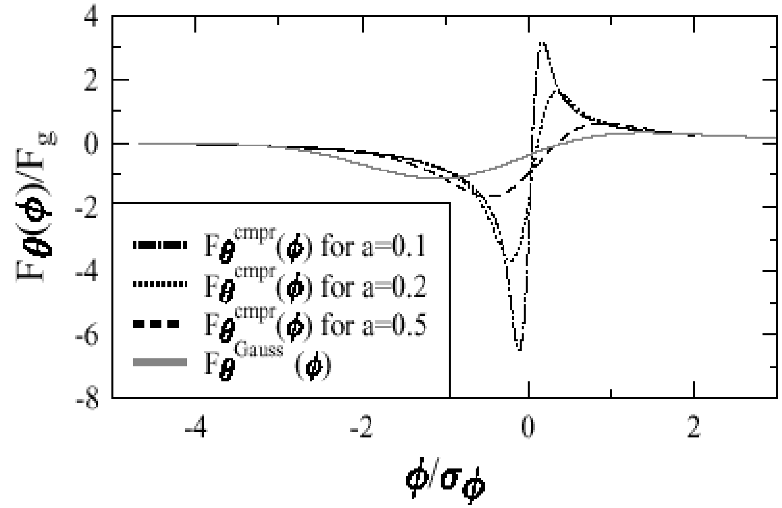


After compressing with rf curvature in the input longitudinal phase space.



Corresponding density profile ($a \equiv R_{56}\sigma_\delta^{un}/\sigma_z$).

Local charge concentration enhances the CSR force.



THEORY OF CSR-INDUCED MICROBUNCHING

[S. Heifets et al., PRST-AB 5, 064401 (2002); Z. Huang and K.-J. Kim (submitted)]

Underlying theme:
The beam's longitudinal phase space carries high-frequency "seed" modulations arising from some source, which are then amplified during bunch compression.

Objective: Calculate gain length of instability, *especially in LCLS compressor*.

Method: Invoke a linearized Vlasov approach with simplifications:

- Dipole length small compared to dipole spacing.
- Steady-state CSR wake (transients more relevant at longer wavelength).
- No shielding, transverse beam size, collective transverse forces.

For large transverse size and zero compression:

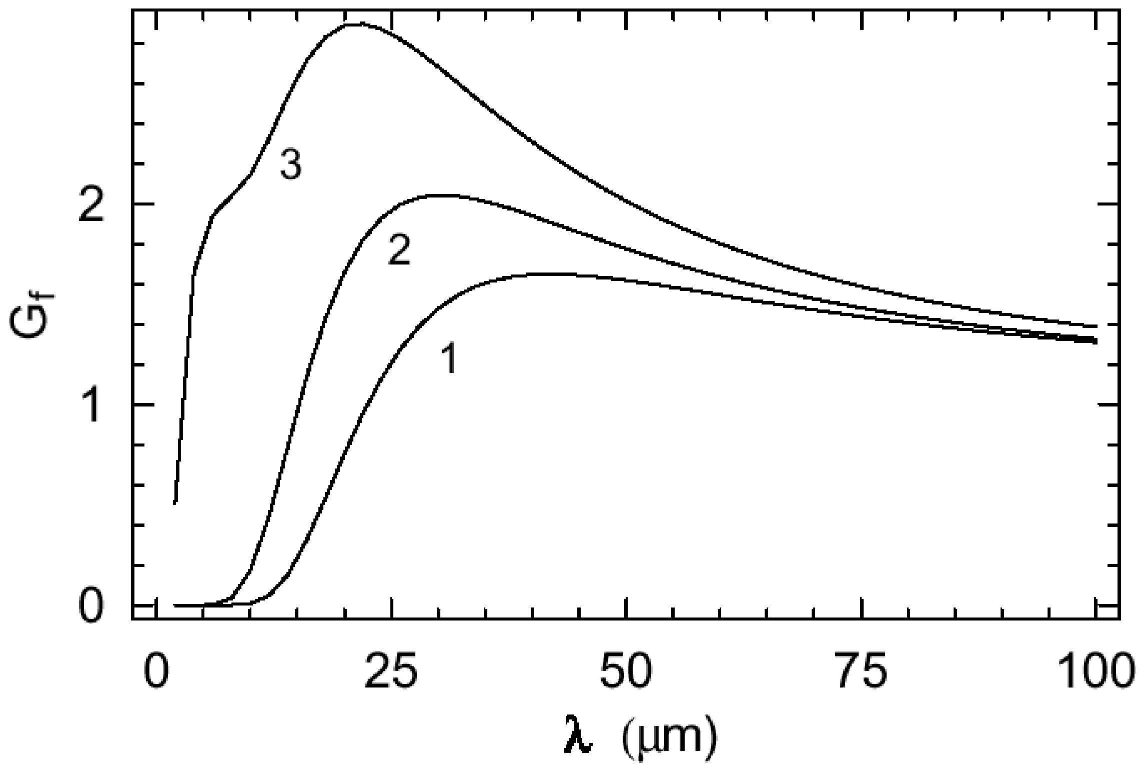
[Saldin, et al., Proc. 2000 FEL Conf. (in press)]

$$G \propto \left(\frac{I\left|R_{56}\right|}{\gamma} \right)^2 \frac{R^{4/3}}{\left(\sigma_x L_D\right)^{8/3}} .$$

RESULTS OF THEORY FOR LCLS COMPRESSOR

Beam energy = 4.54 GeV; Bunch charge = 1 nC;
Bunch length starts at 195 μm, ends at 23 μm.

Gain vs. Perturbation Wavelength at 2nd Compressor Entrance

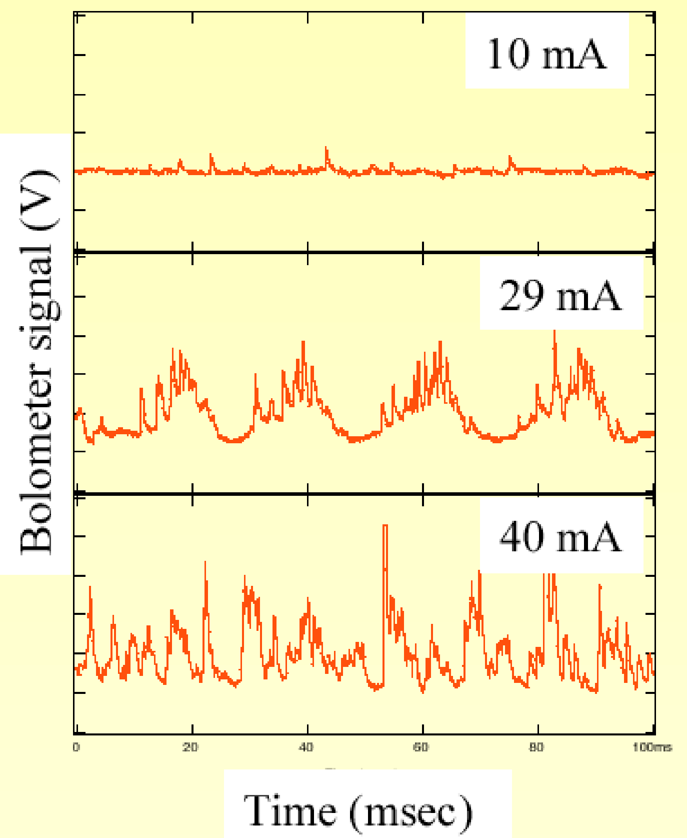


1: $\sigma_\delta^{un} = 3\cdot 10^{-5}$
$\varepsilon = 1$ μm

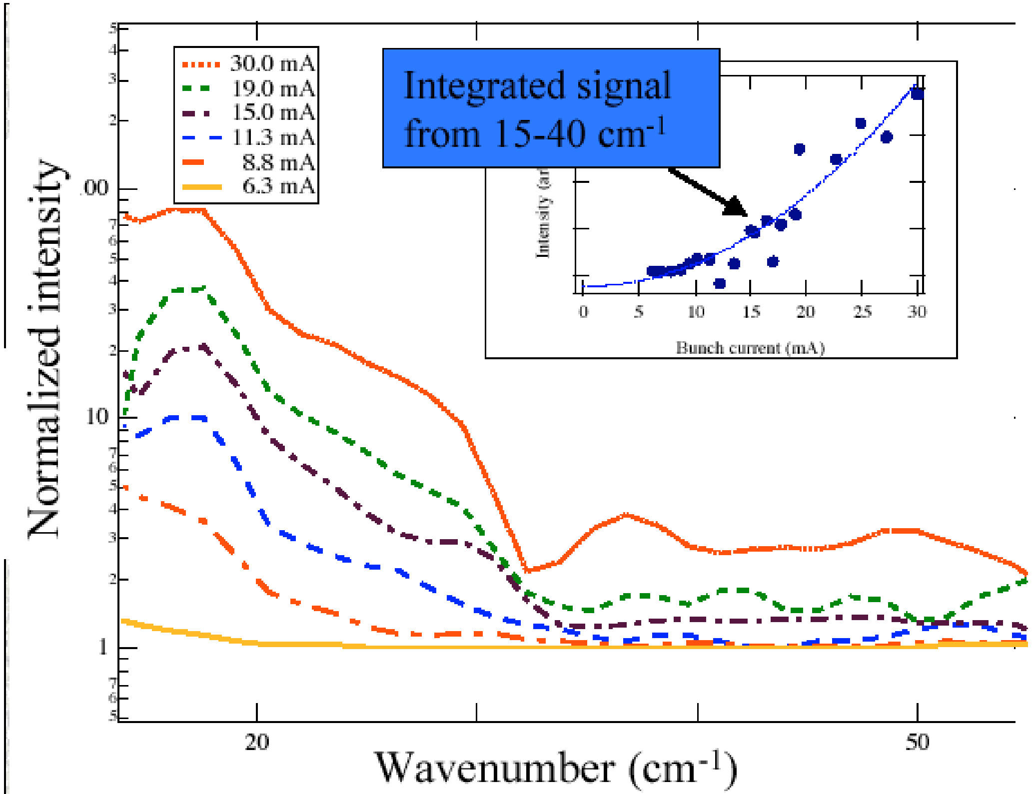
2: $\sigma_\delta^{un} = 3\cdot 10^{-5}$
$\varepsilon = 0$

3: $\sigma_\delta^{un} = 3\cdot 10^{-6}$
$\varepsilon = 1$ μm

COHERENT FAR-IR BURSTS AT ALS



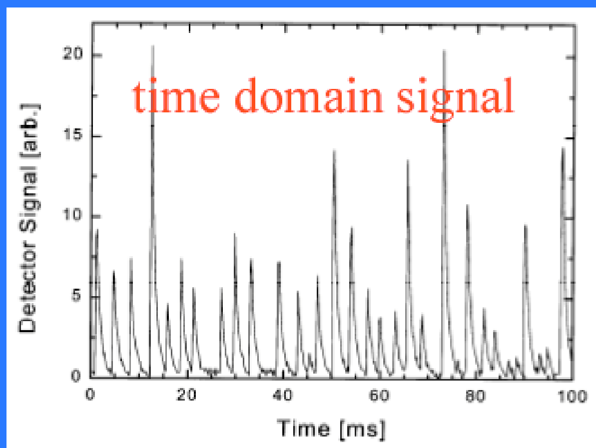
Bursts of far-IR CSR observed on a bolometer. Threshold depends on beam energy, bunch length, energy spread, and SR wavelength.

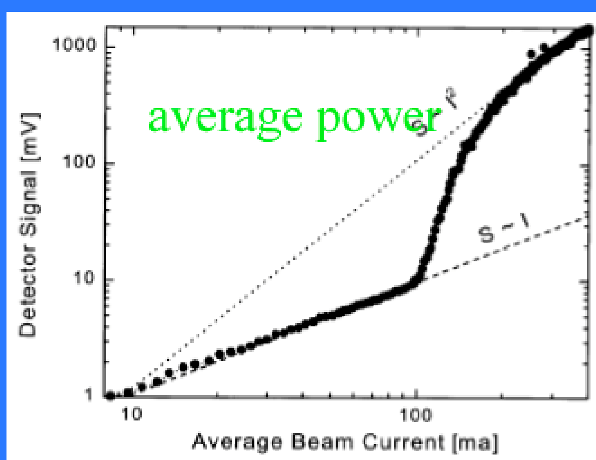


FTIR spectral data shows strong increase below 35 cm^{-1} (300 microns). Integrated signal shows a quadratic increase, confirming coherent radiation.

MICROBUNCHING AND CSR BURSTS IN RINGS

NSLS-VUV Ring (G. Carr, et. al., NIMA 463 (2001) 387-392)

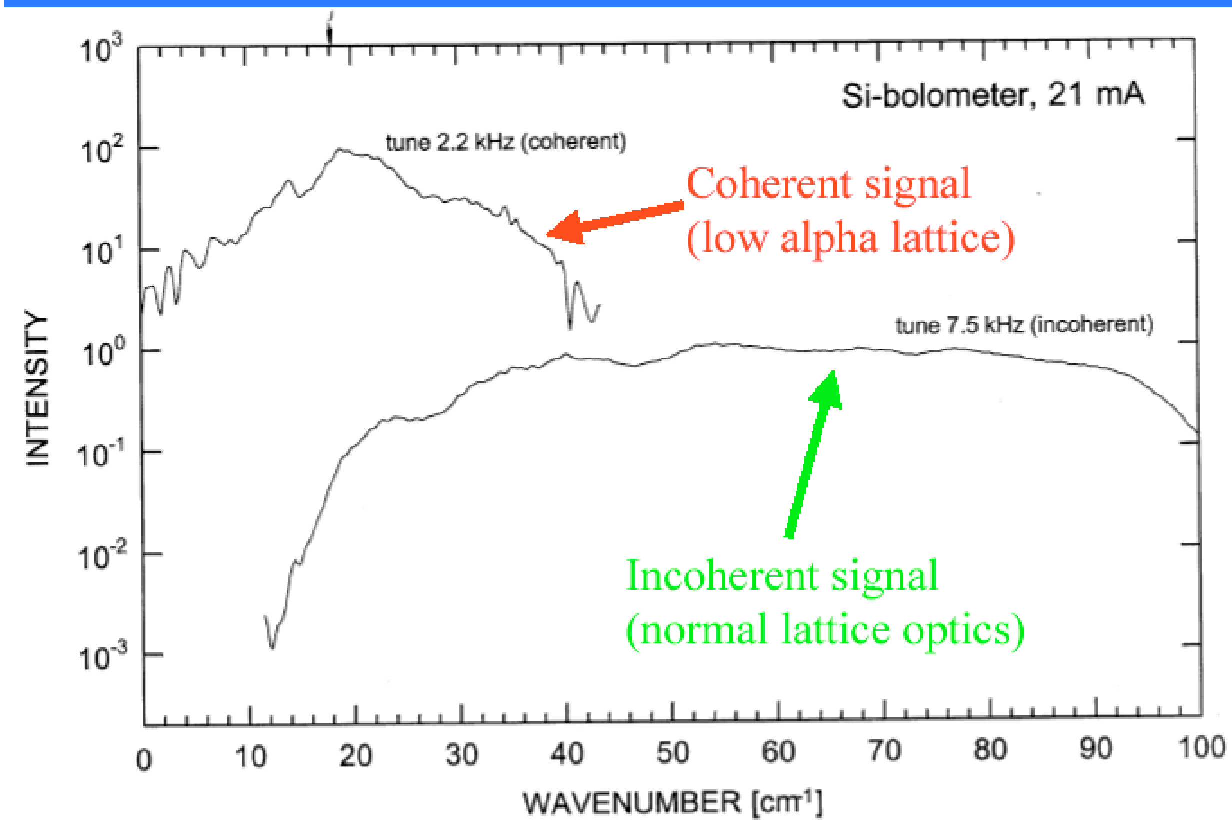




Similar behavior observed at:
- SURF-II/NIST (U. Arp, et. al., Phys. Rev. STAB, **4**, 054401, 2001)
- MAXLab/MAX-I (Å. Anderssen. et. al., Proc. SPIE 3775, 1999, 77.)
- Bessy-II (Abo-Bakr, et. al., submitted to PRL, Oct. 2001., EPAC 2000.)

Bursting instability appears to be fairly universal. Is this a limit for achieving steady coherent emission?

STABLE CSR AT BESSY-II



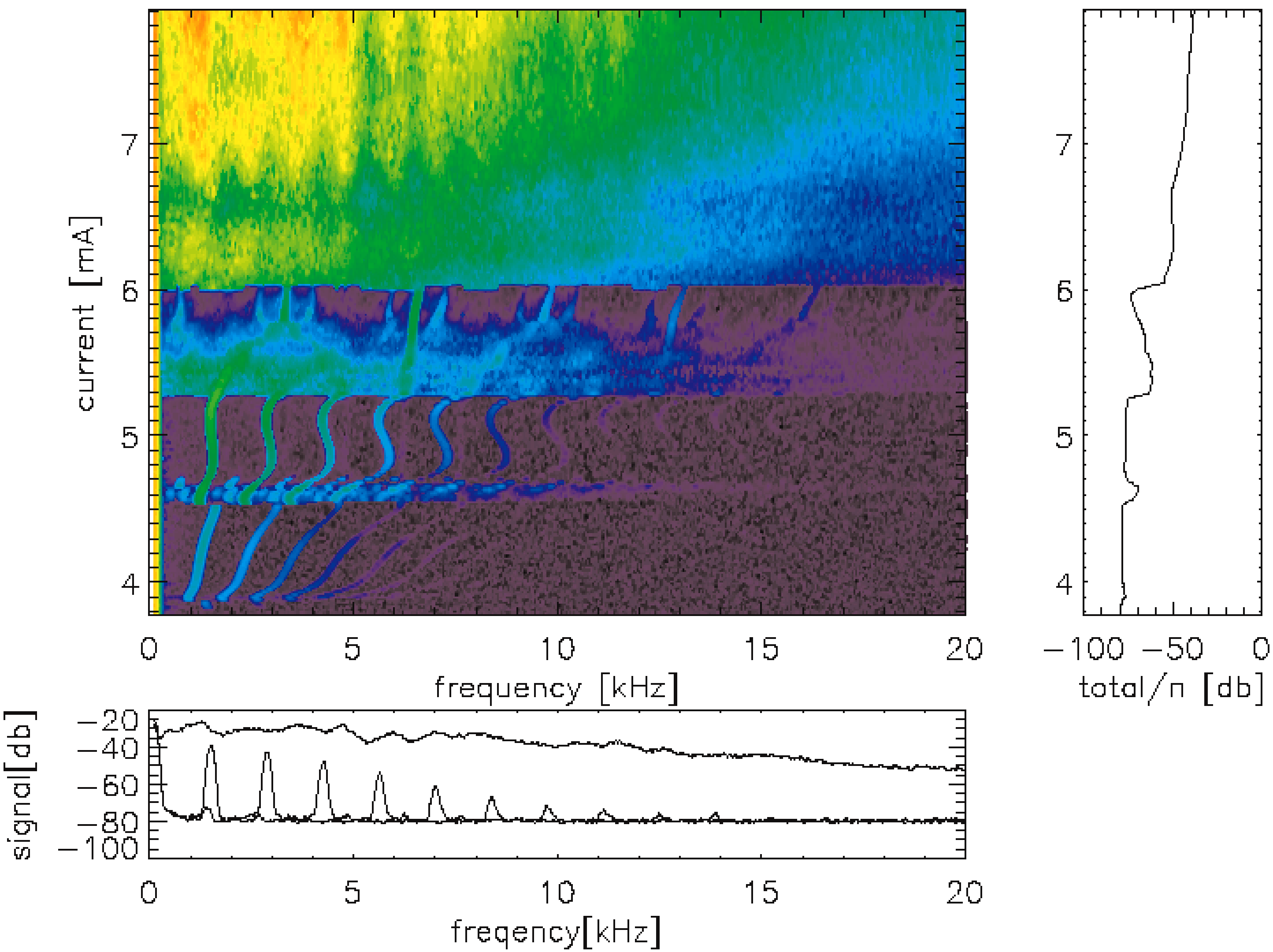
Spectrum of stable coherent and incoherent emission measured at BESSY-II (to appear in Phys. Rev. Lett.)

BESSY-II has demonstrated stable CSR for the first time.

-achieved using low momentum compaction lattice

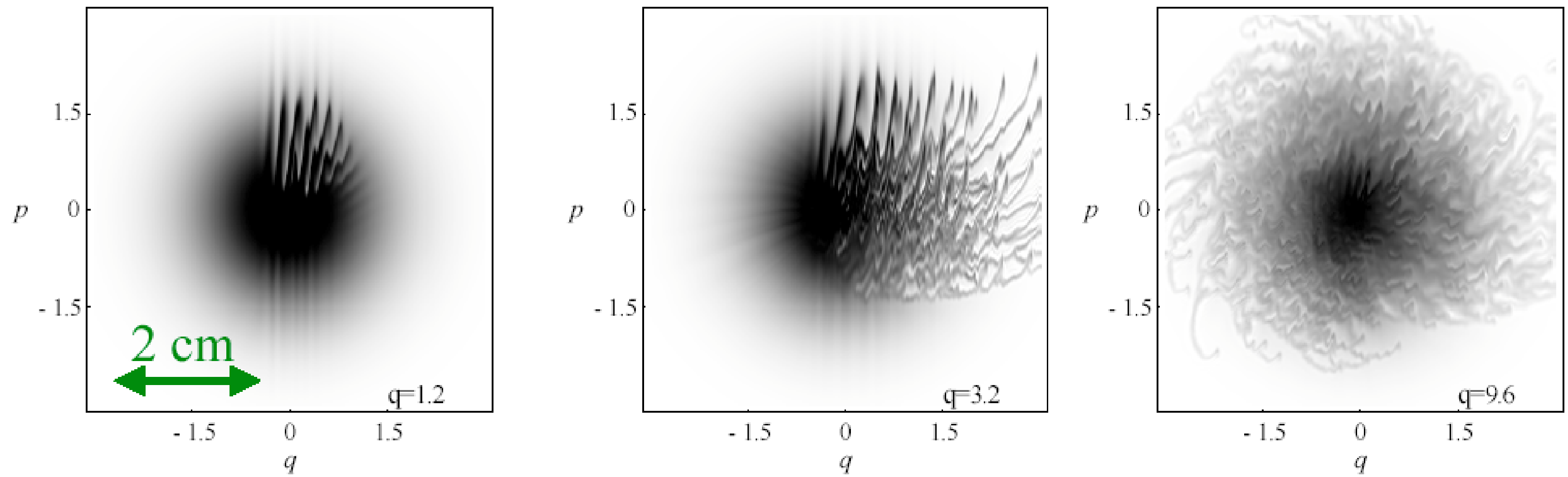
-gains of 1000-5000 observed

PERIODIC TO CHAOTIC BURSTING IN SINGLE-BUNCH MODE (at BESSY II)

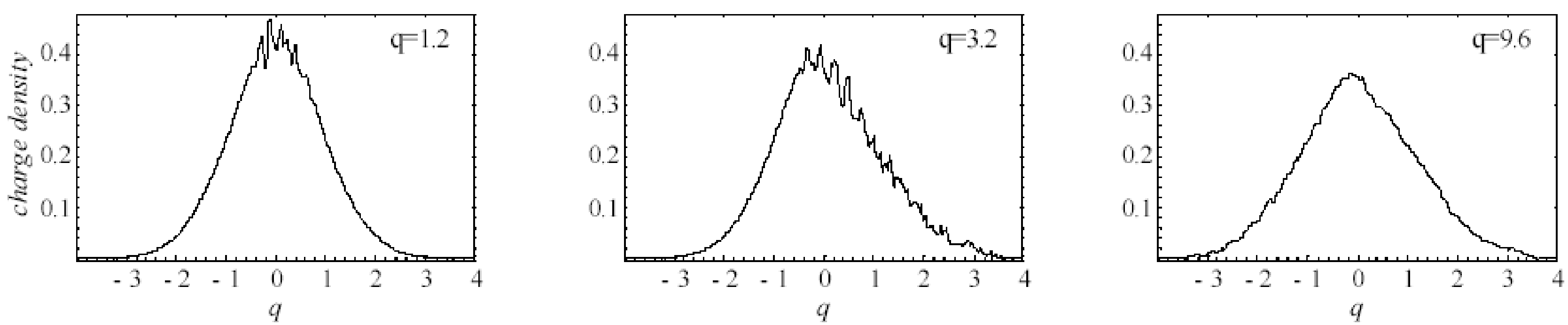


BURST SIMULATIONS

Density Plots in Phase space

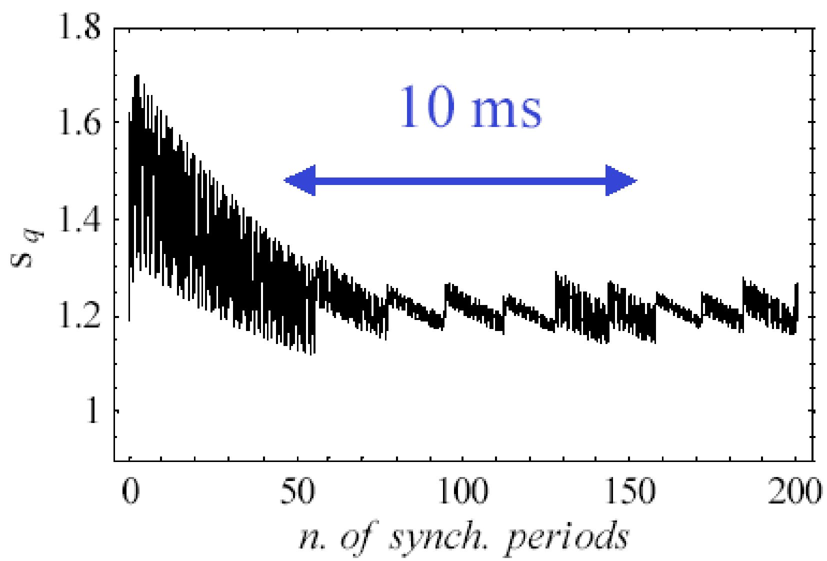


Charge Distribution

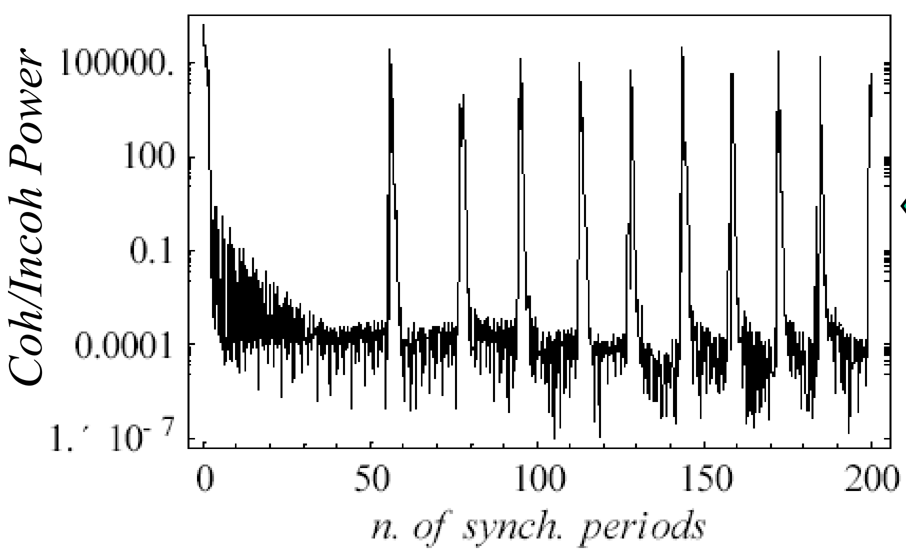


Results of Marco Venturini, Bob Warnock SLAC

BURST SIMULATIONS (cont.)



Bunch length vs. time exhibits sawtooth behavior (i.e. relaxation oscillations)



Ratio of coherent/incoherent power during a burst

SOME PERSONAL IMPRESSIONS…

- Several "interesting developments" over the past 7 years:
  - Realization that CSR can ruin beam quality, as seen in experiments
  - Development of CSR codes (varied success – initial conditions are crucial!)
  - Improved analytic understanding of near-field cancellations
  - Surprise! – microbunch instability – but how important are other wakefields?

- Topics for future work:
  - Capitalize on cancellation of local interactions to develop
    - fast, comprehensive code including effective radial force
    - more comprehensive Vlasov treatment of microbunch instability
  - Develop "cure" of microbunch instability
    - compatible designs of bunch compressors
    - transform to flat beam and compress?
  - Or, "make lemonade" (e.g., the Shintake brew from 2002 CSR Workshop)
    - seed electron beam with a periodic modulation,
    - use microbunch instability to amplify modulation,
    - pass amplified modulations through a FEL system.
  - Or, build a ring-based CSR source? – Let's see!